



TIME-AVERAGED SHADOW MOIRÉ METHOD FOR STUDYING VIBRATIONS

BY

Y. Y. HUNG, C. Y. LIANG, J. D. HOVANESIAN AND A. J. DURELLI

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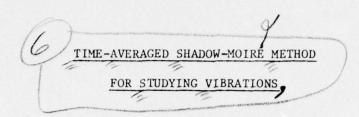
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Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli

ABSTRACT

A time-averaged shadow-moiré method is presented which permits the determination of the amplitude distribution of the deflection of a plate in steady state vibration. No stroboscope is required and the recording is done statically. The method is less sensitive than holographic methods and is therefore suitable for studying relatively large amplitudes.

Previous Technical Reports to the Office of Naval Research

- A. J. Durelli, "Development of Experimental Stress Analysis Methods to Determine Stresses and Strains in Solid Propellant Grains"--June 1962. Developments in the manufacturing of grain-propellant models are reported. Two methods are given: a) cementing routed layers and b) casting.
- 2. A. J. Durelli and V. J. Parks, "New Method to Determine Restrained Shrinkage Stresses in Propellant Grain Models"--October 1962. The birefringence exhibited in the curing process of a partially restrained polyurethane rubber is used to determine the stress associated with restrained shrinkage in models of solid propellant grains partially bonded to the case.
- 3. A. J. Durelli, "Recent Advances in the Application of Photoelasticity in the Missile Industry"--October 1962.

 Two- and three-dimensional photoelastic analysis of grains loaded by pressure and by temperature are presented. Some applications to the optimization of fillet contours and to the redesign of case joints are also included.
- 4. A. J. Durelli and V. J. Parks, "Experimental Solution of Some Mixed Boundary Value Problems"--April 1964.

 Means of applying known displacements and known stresses to the boundaries of models used in experimental stress analysis are given. The application of some of these methods to the analysis of stresses in the field of solid propellant grains is illustrated. The presence of the "pinching effect" is discussed.
- 5. A. J. Durelli, "Brief Review of the State of the Art and Expected Advance in Experimental Stress and Strain Analysis of Solid Propellant Grains"-- April 1964.

 A brief review is made of the state of the experimental stress and strain analysis of solid propellant grains. A discussion of the prospects for the next fifteen years is added.
- A. J. Durelli, "Experimental Strain and Stress Analysis of Solid Propellant Rocket Motors"--March 1965.

 A review is made of the experimental methods used to strain-analyze solid propellant rocket motor shells and grains when subjected to different loading conditions. Methods directed at the determination of strains in actual rockets are included.
- 7. L. Ferrer, V. J. Parks and A. J. Durelli, "An Experimental Method to Analyze Gravitational Stresses in Two-Dimensional Problems"--October 1965. Photoelasticity and moiré methods are used to solve two-dimensional problems in which gravity-stresses are present.

- 8. A. J. Durelli, V. J. Parks and C. J. del Rio, "Stresses in a Square Slab Bonded on One Face to a Rigid Plate and Shrunk"--November 1965.

 A square epoxy slab was bonded to a rigid plate on one of its faces in the process of curing. In the same process the photoelastic effects associated with a state of restrained shrinkage were "frozen-in."

 Three-dimensional photoelasticity was used in the analysis.
- 9. A. J. Durelli, V. J. Parks and C. J. del Rio, "Experimental Determination of Stresses and Displacements in Thick-Wall Cylinders of Complicated Shape"--April 1966.

 Photoelasticity and moiré are used to analyze a three-dimensional rocket shape with a star shaped core subjected to internal pressure.
- 10. V. J. Parks, A. J. Durelli and L. Ferrer, "Gravitational Stresses Determined Using Immersion Techniques"--July 1966.

 The methods presented in Technical Report No. 7 above are extended to three-dimensions. Immersion is used to increase response.
- 11. A. J. Durelli and V. J. Parks. "Experimental Stress Analysis of Loaded Boundaries in Two-Dimensional Second Boundary Value Problems" -- February 1967.

 The pinching effect that occurs in two-dimensional bonding problems, noted in Reports 2 and 4 above, is analyzed in some detail.
- 12. A. J. Durelli, V. J. Parks, H. C. Feng and F. Chiang, "Strains and Stresses in Matrices with Inserts,"-- May 1967.

 Stresses and strains along the interfaces, and near the fiber ends, for different fiber end configurations, are studied in detail.
- 13. A. J. Durelli, V. J. Parks and S. Uribe, "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load,"--June 1967.

 Two-dimensional photoelasticity was used to study various elliptical ends to a slot, and determine which would give the lowest stress concentration for a load normal to the slot length.
- 14. A. J. Durelli, V. J. Parks and Han-Chow Lee, "Stresses in a Split Cylinder Bonded to a Case and Subjected to Restrained Shrinkage,"--January 1968.

 A three-dimensional photoelastic study that describes a method and shows results for the stresses on the free boundaries and at the bonded interface of a solid propellant rocket.
- 15. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--August 1968.

 This report has been written following a trip conducted by the author through several European countries. A list is given of many of the laboratories doing important experimental stress analysis work and of the people interested in this kind of work. An attempt has been made to abstract the main characteristics of the methods used in some of the countries visited.

- 16. V. J. Parks, A. J. Durelli and L. Ferrer, "Constant Acceleration Stresses in a Composite Body"--October 1968.

 Use of the immersion analogy to determine gravitational stresses in two-dimensional bodies made of materials with different properties.
- 17. A. J. Durelli, J. A. Clark and A. Kochev, "Experimental Analysis of High Frequency Stress Waves in a Ring"--October 1968.

 A method for the complete experimental determination of dynamic stress distributions in a ring is demonstrated. Photoelastic data is supplemented by measurements with a capacitance gage used as a dynamic lateral extensometer.
- 18. J. A. Clark and A. J. Durelli, "A Modified Method of Holographic Interferometry for Static and Dynamic Photoelasticity"--April 1968. A simplified absolute retardation approach to photoelastic analysis is described. Dynamic isopachics are presented.
- 19. J. A. Clark and A. J. Durelli, "Photoelastic Analysis of Flexural Waves in a Bar"--May 1969.

 A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.
- J. A. Clark and A. J. Durelli, "Optical Analysis of Vibrations in Continuous Media"--June 1969.

 Optical methods of vibration analysis are described which are independent of assumptions associated with theories of wave propagation. Methods are illustrated with studies of transverse waves in prestressed bars, snap loading of bars and motion of a fluid surrounding a vibrating bar.
- 21. V. J. Parks, A. J. Durelli, K. Chandrashekhara and T. L. Chen, "Stress Distribution Around a Circular Bar, with Flat and Spherical Ends, Embedded in a Matrix in a Triaxial Stress Field"--July 1969.

 A Three-dimensional photoelastic method to determine stresses in composite materials is applied to this basic shape. The analyses of models with different loads are combined to obtain stresses for the triaxial cases.
- 22. A. J. Durelli, V. J. Parks and L. Ferrer, "Stresses in Solid and Hollow Spheres Subjected to Gravity or to Normal Surface Tractions"-- October 1969.

 The method described in Report No. 10 above is applied to two specific problems. An approach is suggested to extend the solutions to a class of surface traction problems.
- 23. J. A. Clark and A. J. Durelli, "Separation of Additive and Subtractive Moiré Patterns"--December 1969. A spatial filtering technique for adding and subtracting images of several gratings is described and employed to determine the whole field of Cartesian shears and rigid rotations.

- 24. R. J. Sanford and A. J. Durelli, "Interpretation of Fringes in Stress-Holo-Interferometry"--July 1970.

 Errors associated with interpreting stress-holo-interferometry patterns as the superposition of isopachics (with half order fringe shifts) and isochromatics are analyzed theoretically and illustrated with computer generated holographic interference patterns.
- 25. J. A. Clark, A. J. Durelli and P. A. Laura, "On the Effect of Initial Stress on the Propagation of Flexural Waves in Elastic Rectangular Bars"--December 1970.

 Experimental analysis of the propagation of flexural waves in prismatic, elastic bars with and without prestressing. The effects of prestressing by axial tension, axial compression and pure bending are illustrated.
- A. J. Durelli and J. A. Clark, "Experimental Analysis of Stresses in a Buoy-Cable System Using a Birefringent Fluid"--February 1971.

 An extension of the method of photoviscous analysis is presented which permits quantitative studies of strains associated with steady state vibrations of immersed structures. The method is applied in an investigation of one form of behavior of buoy-cable systems loaded by the action of surface waves.
- 27. A. J. Durelli and T. L. Chen, "Displacements and Finite-Strain Fields in a Sphere Subjected to Large Deformations"--February 1972.

 Displacements and strains (ranging from 0.001 to 0.50) are determined in a polyurethane sphere subjected to several levels of diametral compression. A 500 lines-per-inch grating was embedded in a meridian plane of the sphere and moiré effect produced with a non-deformed master. The maximum applied vertical displacement reduced the diameter of the sphere by 27 per cent.
- A. J. Durelli and S. Machida, "Stresses and Strain in a Disk with Variable Modulus of Elasticity"--March 1972.

 A transparent material with variable modulus of elasticity has been manufactured that exhibits good photoelastic properties and can also be strain analyzed by moiré. The results obtained suggests that the stress distribution in the homogeneous disk. It also indicates that the strain fields in both cases are very different, but that it is possible, approximately, to obtain the stress field from the strain field using the value of E at every point, and Hooke's law.
- 29. A. J. Durelli and J. Buitrago, "State of Stress and Strain in A Rectangular Belt Pulled Over a Cylindrical Pulley"--June 1972.

 Two- and three-dimensional photoelasticity as well as electrical strain gages, dial gages and micrometers are used to determine the stress distribution in a belt-pulley system. Contact and tangential stress for various contact angles and friction coefficients are given.

- 30. T. L. Chen and A. J. Durelli, "Stress Field in a Sphere Subjected to Large Deformations"--June 1972.

 Strain fields obtained in a sphere subjected to large diametral compressions from a previous paper were converted into stress fields using two approaches. First, the concept of strain-energy function for an isotropic elastic body was used. Then the stress field was determined with the Hookean type natural stress-natural strain relation. The results so obtained were also compared.
- 31. A. J. Durelli, V. J. Parks and H. M. Hasseem, "Helices Under Load"-July 1973.

 Previous solutions for the case of close coiled helical springs and for
 helices made of thin bars are extended. The complete solution is
 presented in graphs for the use of designers. The theoretical development
 is correlated with experiments.
- 32. T. L. Chen and A. J. Durelli, "Displacements and Finite Strain Fields in a Hollow Sphere Subjected to Large Elastic Deformations"--September 1973. The same methods described in No. 27, were applied to a hollow sphere with an inner diameter one half the outer diameter. The hollow sphere was loaded up to a strain of 30 per cent on the meridian plane and a reduction of the diameter by 20 per cent.
- 33. A. J. Durelli, H. H. Hasseem and V. J. Parks, "New Experimental Method in Three-Dimensional Elastostatics"--December 1973.

 A new material is reported which is unique among three-dimensional stress-freezing materials, in that, in its heated (or rubbery) state it has a Poisson's ratio which is appreciably lower than 0.5. For a loaded model, made of this material, the unique property allows the direct determination of stresses from strain measurements taken at interior points in the model.
- 34. J. Wolak and V. J. Parks, "Evaluation of Large Strains in Industrial Applications"--April 1974.

 It was shown that Mohr's circle permits the transformation of strain from one axis of reference to another, irrespective of the magnitude of the strain, and leads to the evaluation of the principal strain components from the measurement of direct strain in three directions.
- 35. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--April 1975.

 Continuation of Report No. 15 after a visit to Belgium, Holland, Germany, France, Turkey, England and Scotland.
- 36. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Linear and Non-linear Elastic and Plastic Strains in a Plate with a Big Hole Loaded Axially in its Plane"--July 1975.

 Strain analysis of the ligament of a plate with a big hole indicates that both geometric and material non-linearity may take place. The strain concentration factor was found to vary from 1 to 2 depending on the level of deformation.

- 37. A. J. Durelli, V. Pavlin, J. O. Bühler-Vidal and G. Ome, "Elastostatics of a Cubic Box Subjected to Concentrated Loads"--August 1975.

 Analysis of experimental strain, stress and deflection of a cubic box subjected to concentrated loads applied at the center of two opposite faces. The ratio between the inside span and the wall thickness was varied between approximately 5 and 121.
- 38. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Elastostatics of Cubic Boxes Subjected to Pressure"--March 1976.

 Experimental analysis of strain, stress and deflections in a cubic box subjected to either internal or external pressure. Inside span-to-wall thickness ratio varied from 5 to 14.

Introduction

Besides its uses in evaluating surface depth contours (1-3) and in determining surface deformations, (4-8) the shadow moiré method was also used to study nodal patterns in vibrating plates. (9) However, to study amplitude distribution, a stroboscopic method was required. (10) This paper presents a time-averaged shadow moiré method whereby the distribution of the amplitude of the plate deflection can be obtained without stroboscopic equipment. In the method, the shadow moiré contour fringes of an object in steady state vibration is photographed with an exposure time equal to one or several vibrational periods. The processed photograph produces a time-averaged fringe pattern depicting the vibrational amplitudes.

The time-averaged effect utilized in the present paper was first applied by Powell and Stetson to vibration studies using holography. (11) The hologram they obtained recorded the position of a steady state vibrating object with a long exposure time compared to the vibrational period. They showed that in the reconstruction of this hologram a time-averaged interference fringe pattern was produced which measured the vibrational amplitudes. The fringe pattern could be represented mathematically by a zero-order Bessel function. The time-averaged effect was later applied to study vibrations using projected gratings methods. (12,13) However, the two techniques are on opposite extremes in the range of sensitivities. While the time-averaged holography is extremely sensitive being able to measure vibrational amplitudes of the order of wavelengths of light, the projected gratings methods are rather insensitive. Therefore, there is a large gap in the sensitivity range between the two techniques. The present method, though still unable to bridge the entire gap, has extended the sensitivity of the projected gratings methods.

Description of the Method

The method uses a standard shadow moiré arrangement as shown in Fig. 1. A master grating with lines running parallel to the y-axis is located in front of and close to the object to be studied. A collimated beam of light illuminates the grating at an angle θ to the y,z-plane, and a distant camera views the grating normally.

If the object surface to be studied is flat, it is carefully positioned so that the stationary moiré contour fringes are null. Then with the object being excited to vibrate steadily, a time-dependent moiré fringe pattern due to the vibrational displacements is observed. If the time-varying fringe pattern is recorded by the camera with an exposure time of one or several vibrational periods, the processed photograph will yield a time-averaged fringe pattern which is related to the vibrational amplitudes by a zero-order Bessel function.

For objects which are not flat, it is not possible to null the stationary contour fringes. In this case, it is advisable to use a rather dense initial contour fringe pattern deliberately introduced by slightly tilting the object about an axis perpendicular to the viewing direction. This initial fringe pattern is then used as a carrier which is modulated by the vibrational amplitudes. The time-averaged fringes formed by the modulated carrier also depict the amplitude distribution of the vibration.

If the vibrational frequency is high enough (20 Hz or higher), the image retaining nature of the eye can do the time averaging. Hence, real time averaged moiré fringes may be observed by the naked eye.

Theory of the Method

Assume that the intensity transmittance of the master grating is sinusoidal and represented by:

$$T(x,y) = 1 + \sin \frac{2\pi}{p} x \tag{1}$$

where p is the grating pitch. The obliquely illuminating beam casts a shadow of the grating onto the object surface. Under the condition that the grating is coarse enough for diffraction effects to be neglected, it can be shown that the intensity distribution $I_g(x,y)$ of the shadow is: (1)

$$I_{S}(x,y) = k \left[1 + \sin \frac{2\pi}{p} (x - z \tan \theta)\right]$$
 (2)

where k is a constant depending on the scattering attenuation of the object surface and z is the surface elevation.

Since the camera views the shadow through the master ruling, the intensity of the image $I_{o}(x,y)$ detected is the product of the shadow intensity and the grating transmittance and is given by:

$$I_{O}(x,y) = I_{S}(x,y) \cdot T(x,y)$$
 (3)

Expansion of the above equation yields

$$I_{O}(x,y) = k \left\{ 1 + \sin \frac{2\pi}{p} x + \sin \frac{2\pi}{p} (x - z \tan \theta) + \frac{1}{2} \cos \frac{2\pi}{p} (2 x - z \tan \theta) + \frac{1}{2} \cos (\frac{2\pi}{p} z \tan \theta) \right\}$$
(4)

All the terms on the right-hand side of the above equation are of high spatial frequency which will average to zero, except the following:

$$I_o(x,y) = k \left\{ 1 + \frac{1}{2} \cos \frac{2\pi}{p} z \tan \theta \right\}$$
 (5)

This equation predicts the formation of moiré fringes depicting z, the contour of the surface.

With the surface undergoing steady state sinusoidal oscillation, z is given by:

$$z = z_0 + A(x,y) \cdot \cos (\omega t + \phi)$$
 (6)

where z_0 is the stationary contour of the object surface; A(x,y) is the amplitude distribution of the vibrating surface; ω is the circular frequency, and ϕ represents the arbitrary initial phase. Thus, z in Eq. (5) is time-dependent and $I_0(x,y)$ should be replaced by $I_0(x,y,t)$. If the photographic film in the camera is exposed to $I_0(x,y,t)$ the exposure on the film is an integration of $I_0(x,y,t)$ over the exposure time which can be adjusted to an integral multiple of the vibrational periods. The integrated intensity is $I_1(x,y)$ given by:

$$I_{i}(x,y) = \int_{0}^{nT} I_{o}(x,y,t) dt$$
 (7)

where nT is the exposure time; n is an integer and T the vibrational period.

Techniques

1. For flat objects:

For a flat object, it is possible to null the stationary contour fringes by positioning the object so that $z_0 = \text{constant}$ (i.e., the flat surface is parallel to the plane of the grating). Then Eq. (6) can be integrated to yield:

$$I_o(x,y) = nT \cdot k \cdot \left\{ 1 + \frac{1}{2} \cdot \cos(\frac{2\pi}{p} z_o) \cdot J_o \left[\frac{2\pi}{p} \tan\theta \cdot A(x,y) \right] \right\}$$
 (8)

The above equation indicates a fringe pattern depicted by a zero-order Bessel function containing A(x,y), the vibrational amplitude distribution, in its argument. Dark fringes occur when the Bessel function attains its minimum, i.e.

$$\frac{2\pi}{p} \tan\theta \cdot A(x,y) = B_{j}$$
 (9)

where B_j is the argument of the jth minimum value of J_o . By rewriting Eq. (9) as

$$A(x,y) = \frac{B_j}{2\pi} \left(\frac{p^j}{\tan \theta} \right)$$
 (10)

and defining $\frac{B_j}{2\pi} = N$ as the fringe order, then the amplitude distribution A(x,y) can be determined by the following equation

$$A(x,y) = N(\frac{p}{\tan \theta}) \tag{11}$$

where N = 0.610, 1.62, 2.62, 3.62, 4.62, . . .

2. For arbitrarily curved surfaces

For curved surface it is not possible to null the stationary contours. In this case, relatively high density fringes are deliberately introduced by slightly rotating (say α) the object about x-axis. z in Eq. (6) becomes

$$z = (z_0 + \alpha y) + A(x,y) \cdot \cos(\omega t + \phi)$$
 (12)

Equation (9) is then substituted into Eq. (7). Integration of the resulting equation yields

$$I_a(x,y) = 1 + \frac{1}{2} \cos \frac{2\pi}{p} (z_0 + \alpha y) \cdot J_0 \left[\frac{2\pi}{p} \tan \theta \cdot A(x,y) \right]$$
 (13)

Equation (13) is an expression where a high frequency term is amplitude modulated by $J_0[\frac{2\pi}{p}\tan\theta\cdot A(x,y)]$. In this case the nulling of the high frequency term (carrier) is identified as moiré fringes which occur when the Bessel function is equal to zero, i.e., the argument is equal to the roots of the function. It can be shown, by following a similar analysis as in the last section, that the amplitude distribution A(x,y) can be determined by Eq. (10). For this case, $N = \frac{1}{2\pi}$ where B_j is the jth root of J_0 , and takes the values of 0.383, 0.879, 1.38, 1.88, 2.38, . . .

While the first technique is only applicable to flat objects, the fringe modulation technique can be used for studying objects of both flat and curved surfaces. Our experience shows that fringes of better visibility were obtained using the first technique. However, the fringe modulation technique is better suited for studying nodal patterns.

Similar results will be obtained if the intensity transmittance of the grating is a square function with the advantage that a better fringe visibility can be achieved.

Experiments

A circular rubber membrane of 10 cm diameter clamped along its boundary was selected for demonstration. It was excited into vibration from behind by a loud speaker driven by a wave generator. A one-way grating of 20 lines per centimeter was used. The light source was a projector located at a great distance from the object in a direction making an angle $\theta = 60^{\circ}$ with the normal to the plate.

The grating was carefully positioned in respect to the plate to avoid any initial interference. The plate was then excited at its natural frequencies of 100 and 185 Hz. The fringe patterns obtained depicting the amplitude distributions of the vibrational modes are shown in Fig. 2.

To demonstrate the fringe carrier technique, the membrane was slightly tilted to produce the initial fringe carrier shown in Fig. 3(A). The time-averaged moiré fringes formed by the fringe carrier when the membrane was excited to vibrate at 100 Hz and 185 Hz are shown in Fig. 3(B) and Fig. 3(C), respectively.

Conclusion

It has been shown that time-averaged shadow moiré fringes are formed when an object is undergoing steady state oscillation. The fringes are loci of

deflection amplitude interpreted by a zeroth order Bessel function. Owing to the fact that the value of the Bessel function decreases as the value of the argument increases, the fringe visibility drops with the increasing amplitude. This provides a means of identifying nodal areas as well as the fringe orders.

In the projected grating method, it is necessary for the imaging system to resolve the grating projected on the object surface. Therefore, the limit to the fineness of the grating that can be projected and recorded bounds the sensitivity of the method. The present method requires only the moiré fringes to be resolved, thus allowing gratings of higher frequency to be used. Hence, it extends the sensitivity range of the projected grating method.

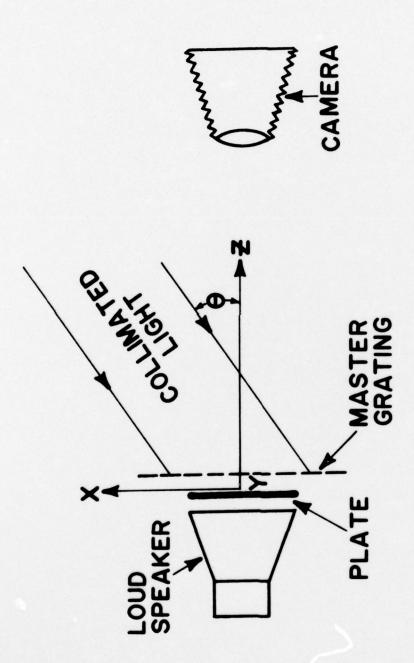
Acknowledgement

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References

- Meadows, D. M., Johnson, W. O. and Allen, J. B., "Generation of Surface Contours by Moiré Patterns," <u>Applied Optics</u>, 9 (4), 942-947, 1970.
- Takasaki, H. "Moiré Topography," <u>Applied Optics</u>, Vol. 9, No. 6, 1476, 1970.
- Chiang, Fu-Pen, "A Shadow-Moiré Method with Two Discrete Sensitivities,"
 Experimental Mechanics, Vol. 15, No. 10, 382, 1975.
- 4. Weller, R. and Shepard, B. M., "Displacement Measurement by Mechanical Interferometry," Proceedings SESA, Vol. 6, No. 11, 35, 1948.
- 5. Theocaris, P. S., "Isopachic Patterns by the Moiré Method," Experimental Mechanics, Vol. 4, No. 6, 153-159, 1968.
- Theocaris, P. S., "Moiré Pattern of Slope Contours in Flexed Plates,"
 Experimental Mechanics, Vol. 6, No. 4, 115-120, 1966.
- 7. Dykes, B. C., "Analysis of Displacements in Large Plates by the Grid-Shadow Moiré Technique," Experimental Stress Analysis and Its Influence on Design," Proc. of the 4th. Int. Conf. on Exp. Str. Analysis, 125-134, 1971.
- 8. Stromondo, A., Katz, A. and Chiang, F.P., "Mapping Large Deflection of Structures by Moiré Method." <u>Proceedings U.S. Army Solid Mechanics</u>
 Symp., Ocean City, MD, October 3-5, 1972.
- 9. Hazell, C. R. and Niven, R. D., "Visualization of Nodes and Antinodes in Vibrating Plates," Experimental Mechanics, 8 (5), 225-231, 1968.
- Theocaris, P. S., "Moiré Fringes in Strain Analysis," Pergamon Press,
 1969, Chap. 7.

- 11. Powell, R. L. and Stetson, K. A., "Interferometric Vibration Analysis by Wavefront Reconstruction," J. Opt. Soc. Am. 55, 1593, 1965.
- 12. Hovanesian, J. D. and Hung Y. Y., "Moiré Contour-Sum, Contour-Difference and Vibrational Analysis of Arbitrary Object," Applied Optics, Vol. 10, No. 12, 1971.
- 13. Vest, C. M. and Sweeney, D. W., "Measurement of Vibrational Amplitude by Modulation of Projected Fringes," <u>Applied Optics</u>, Vol. 11, No. 2, 449, 1972.



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FIGURE 1 SCHEMATIC DIAGRAM OF SHADOW-MOIRÉ METHOD FOR VIBRATION STUDIES

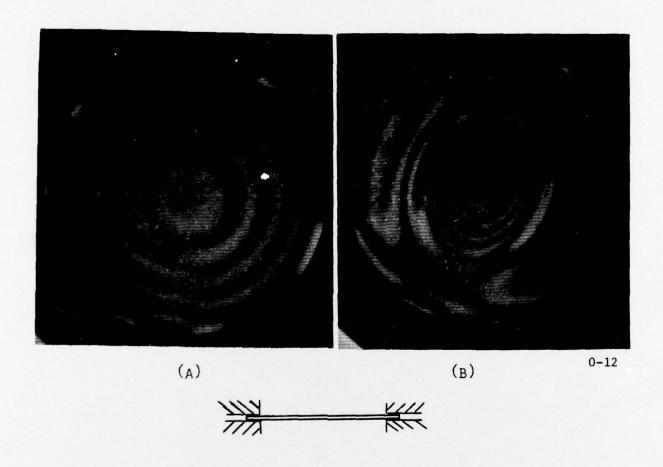
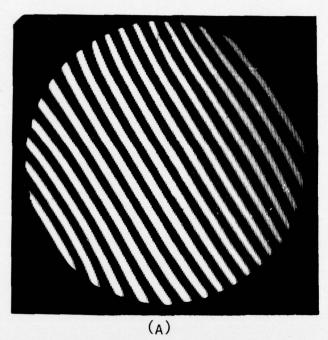


FIGURE 2 FRINGE PATTERN DEPICTING VIBRATIONAL AMPLITUDE OF A RUBBER MEMBRANE



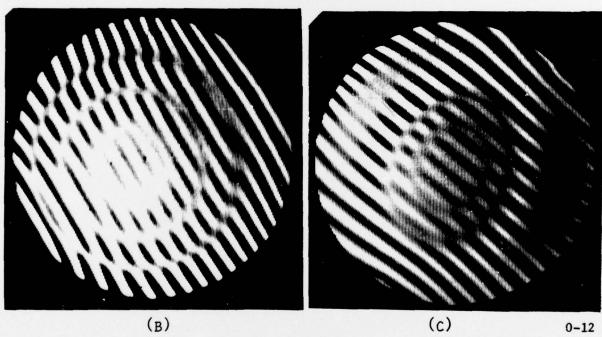


FIGURE 3(A) INITIAL FRINGE CARRIER OF THE CIRCULAR, CLAMPED MEMBRANE

- (B) Moiré Fringe Pattern Depicting Vibrational Amplitude at 100 Hz
- (c) Moiré Fringe Pattern Depicting Vibrational Amplitude at 185 Hz

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A time-averaged shadow-moire method is presented which permits			
	the determination of the amplitude distribution of the deflection		
	of a plate in steady state vibration. No stroboscope is required		
	and the recording is done statically. The method is less sensi-		
	tive than holographic methods and is therefore suitable for		
	studying relatively large amplitudes.		
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